



# FIELDS NOTES

VOLUME 21:3  
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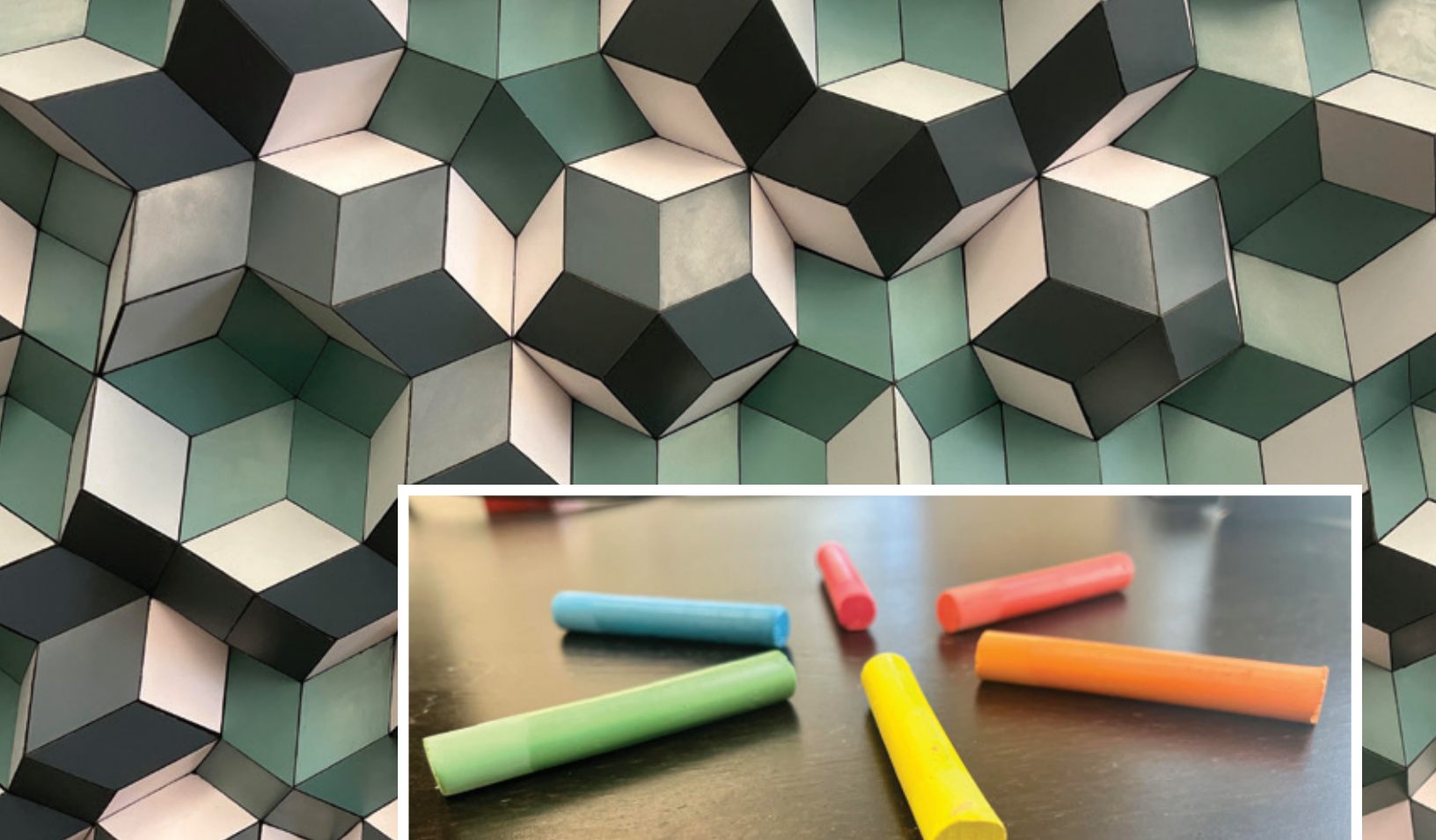
THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES



**Taming of  
the surface:**  
*Fields gets its own  
Wieringa Wall*

**Op-ed:**  
*We need a  
national  
math strategy*

**AI for Good:**  
*How machine  
learning is  
revolutionizing  
medical  
diagnostics*



### MISSION STATEMENT

At the Fields Institute, mathematics research, innovation and education flourish. We foster an inclusive, equitable and collaborative culture where everyone can discover mathematics, and where mathematicians make meaningful contributions to the world.

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**Fields Notes**, The Fields Institute for Research in Mathematical Sciences

Director: **Kumar Murty**

Deputy Director: **Deirdre Haskell**

Managing Editor: **Jordana Feldman**

Contributing Writers: **Pam Brittain, Ross J. Cocks, Lynda Colgan, Daniel Jacobs, Alik Sokolov**

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**FIELDS**



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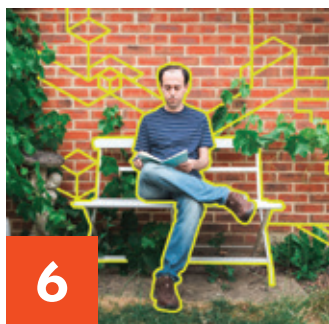
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“ I had heard about the Fields Institute as a world class institute of mathematics for many years. Yet somehow, I never had the chance to visit it for more than a few hours. However, this time, when I visited for a few months, I was able to observe its special qualities at close hand. I was truly impressed by the amount of activity in both pure and applied mathematics, and even more so by combining of the two, which seems to be rare in current mathematics worldwide. ”

- Alex Lubotzky (Weizmann Institute of Science)  
2022 Simons Distinguished Visitor

## UPCOMING OPPORTUNITIES

### Fields Research Fellowships

Faculty members at Principal Sponsoring Universities in Canada can nominate a mathematical scientist for a Fields Research Fellowship for the purpose of collaborative research.

*Application deadlines: January, May 15 and September 15 of each calendar year.*

### Dean's Distinguished Visiting Professorship

A joint program of the Fields Institute and select Principal Sponsoring Universities, the DDVP brings leading international researchers in the mathematical sciences to the Fields Institute to be in residence for a term. Currently, we have two annual positions: one with McMaster University and one with the University of Toronto.

*Rolling application process. Please check the Fields website for more details.*

### CRM-Fields-PIMS Prize

The CRM-Fields-PIMS prize is the premier Canadian award for research achievements in the mathematical sciences. It is awarded jointly by the three Canadian mathematics institutes.

*Nomination deadline: November 1, 2023.*

### The Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education

This award recognizes an early-career educator in Canada who has demonstrated innovation and excellence in promoting mathematics education at the elementary, secondary, college or university level.

*Nomination deadline: December 31, 2023*

# WE NEED A NATIONAL MATH STRATEGY

## Failure to invest in mathematics is a failure to invest in the future



I was speaking with Robert Prichard recently. In 1995, Rob was the President of the University of Toronto, and he was instrumental in getting the Fields Institute to be permanently located in Toronto. He had the vision to see the synergistic influence the Institute and the University would have on each other, propelling each other to greater heights, and the last few decades have certainly proved him to be right!

From his vantage point today as Chair of the Board of the law firm Torys, he fondly remembered those days, and our conversation turned to legacy. He asked me if there were any more Jim Arthurs around. Jim is a distinguished Canadian mathematician who is recognized around the world for his fundamental contributions to the theory of automorphic representations.

I told him there are some incredibly smart people, but if we want to produce more Jim Arthurs, Fields medallists or Abel Prize winners, we need to build a larger funnel and attract people towards mathematics. That requires a strategy. *A national mathematics strategy.*

Building such a strategy is not just a necessity for global

academic leadership; it is a foundational requirement for any country to remain competitive in an increasingly complex and technology-dependent world. If you look at any of the major innovations over the past 50 years, you will notice they all have a mathematical core. Mathematical ideas are like the stem cells of innovation: Artificial intelligence, quantum science, next generation communications (including 5G and 6G) and advanced neuroscience are obvious ones.

But there are also broader applications that have, until recently, been perceived to fall outside the purview of traditional mathematical science. Over the course of the COVID-19 pandemic, we saw the critical role advanced mathematical modelling played in combatting public health risk and informing policy decisions. The same methodology applies to mitigating the effects of climate change and creating smart villages that have the capability to improve quality of life on a mass scale.

Mathematical sciences are not an add-on. They are an essential component of any country's innovation pillars and thus its economic robustness. The pillars of

this strategy are research, training and commercialization. Societies that fail to recognize this necessity, that fail to invest in building mathematical research capacity, do so at their own peril.

One of the main obstacles, however, is a perception that research and innovation are opposites; that mathematics falls squarely in the area of research, and that what we really want more of is innovation. While innovation is encouraged as an engine of economic growth, research is seen as a luxury that a small minority can afford to engage in. Such a point of view evinces a lack of understanding of the process by which knowledge translation occurs.

A good example of this is Canada's outsized role in the current AI boom. Three of the pioneers of modern AI, Geoffrey Hinton, Yoshua Bengio and Richard Sutton, are Canadian-based academics whose research was supported for decades before computational power and large enough datasets caught up with their intuitions and made it possible to implement their ideas. If they had waited for the computational power before developing their ideas on deep learning, we would have missed the boat!

# WE NEED A NATIONAL MATH STRATEGY



Moreover if we want to score a homerun like this, there has to be a sufficiently abundant reservoir of ideas so that we can cast a wide net and yield a rich catch.

Another obstacle is to understand the role of time. If we want to enjoy the fruits of a tree today, the tree should have been planted and nurtured years earlier. Some of the most significant research initiatives can take decades to show results and require investment partners with an appetite for risk and ability to forecast.

We understand the hesitancy for sitting governments and funding agencies to invest in long-term research programs that may exceed their own term limits, or that may fail to provide a quick return on investment. Nevertheless, it is the long-term strategy that is needed to energize and drive a continuous process of innovation.

At Fields, we see research and innovation as part of a single process, each building upon the other. The mathematical science institutes in Canada, including the Fields Institute, the Atlantic

Association for Research in Mathematical Sciences (AARMS), the Centre de Recherches Mathématiques (CRM) and the Pacific Institute of Mathematical Sciences (PIMS), have over several decades demonstrated their ability to mobilize research in targeted areas of mathematical sciences.

We therefore propose the formation of a National Mathematics Secretariat that will be created with representatives of the four mathematical science institutes and will produce a blueprint for a five-year program of research, training and commercialization as part of a National Mathematics Strategy. Following the development of that blueprint, we will then work to refine it in consultation with all major stakeholders in the country.

The mission of this strategy is two-fold. The first is to grow Canada's mathematical sciences capacity so as to be assured of sufficient talent to feed into the innovation strategy. This includes addressing the equity, diversity and inclusion (EDI) disparity early in the training cycle to maximize the talent pipeline. It also involves

interdisciplinary collaboration between mathematicians and the consumers of mathematical ideas.

The second is to drive research, training and commercialization of the applications of mathematical sciences by forging stronger partnerships between academia, industry and government. This can be improved by increasing awareness of the ubiquity of mathematics in all phases of innovation.

After the series of challenging events we have faced over the past decade, from a global pandemic to inflation and climate change, mathematics can be used as a stabilizing force to build more advanced and secure foundations. Times of chaos are a rare opportunity to shape the future in better and progressive ways. We have the scientific talent, infrastructure and experience to be a global leader in these areas. But we need a national mathematics strategy to turn that into a reality.  $\square$

*Kumar Murty is the Director of the Fields Institute.*

# 2023/2024 THEMATIC AND FOCUS PROGRAMS CALENDAR

A list of upcoming events linked to our core programs

**Thematic Program on Operator Algebras and Applications**

July 1 – December 31, 2023

**Twinned Conference on  $C^*$ -Algebras and Tensor Categories**  
November 6 - 10, 2023

**Workshop on Operator Algebras and Applications: Free Probability**  
November 13 - 17, 2023

**Workshop on Operator Algebras and Applications: Non-Commutative Geometry**  
December 4 - 8, 2023

**2023-2024 Operator Algebra Seminar**  
July 1, 2023 - June 30, 2024

**Model Theory of Operator Algebras Reading Seminar**  
July 1, 2023 - June 30, 2024

**Distinguished Lecture Series: Tatiana Shulman**  
October 2 - 6, 2023

**Graduate Course on K-Theory and  $C^*$ -Algebras**  
September 11 - December 8, 2023

**Graduate Course on the Tangent Groupoid in Noncommutative Geometry**  
September 12 - November 30, 2023

**Thematic Program on Randomness and Geometry**

January 1 – June 30, 2024

**Workshop KPZ meets KPZ**  
March 4 - 8, 2024

**Workshop on Dynamics of group actions and Random walks on groups**  
May 13 - 17, 2024

**Graduate Course: Equidistribution and mixing for the geodesic flows of surfaces**  
January 8 - June 30, 2024

**Mini Course: Equidistribution and mixing for the geodesic flows of surfaces**  
January 8 - 19, 2024

**Mini Course: Random walks on groups**  
January 15 - 26, 2024

**Mini Course: Dynamics of group actions**  
January 22 - February 2, 2024

**Mini Course: Yang-Mills**  
January 29 - March 8, 2024

**Mini Course: Random covers of graphs and surfaces**  
February 12 - 23, 2024

**Mini Course: Loewner energy**  
February 26 - March 8, 2024

**Mini Course: Liouville quantum gravity**  
March 11 - 29, 2024 

Visit [fields.utoronto.ca/calendar](https://fields.utoronto.ca/calendar) for full event listings. 

# FMS 2023: CAUCHER BIRKAR




The 2023 Fields Medal Symposium will honour Caucher Birkar. Birkar was awarded the Fields Medal in 2018 “for his proof of the boundedness of Fano varieties and contributions to the minimal model program.”

Birkar is perhaps best known for the development of new techniques and tools for understanding the birational geometry of algebraic varieties and for settling several long-standing problems in the field. In particular, his work has helped prove that all algebraic varieties can be reduced to one of three basic types through birational transformations, bringing order to the associated infinite zoo of polynomial equations. Further he showed that Fano varieties form a neat family that can be defined by a small number of parameters.

His work covers a broad range of topics and classification problems in birational geometry including minimal models, flips, Fano and Calabi-Yau varieties, singularities,

generalized pairs, moduli theory and positive characteristic geometry.

Classification is a theme that has also played out in Birkar’s personal life. His name, which he changed to reflect his challenging path to academic success, can be interpreted as “immigrant mathematician” or “explorer mathematician” in Kurdish. His mathematical abilities landed him at the University of Tehran. He did his graduate studies at the University of Nottingham, UK.

Birkar’s lived experiences, and notable contributions, make him an important advocate for the role of education and scientific research in improving people’s lives and building a better future. 



*Photo credits: Philipp Ammon for Quanta Magazine*



# BOOK REVIEW:

## Fibonacci's *Liber Abaci*

### What the 900-year-old text can still teach us today

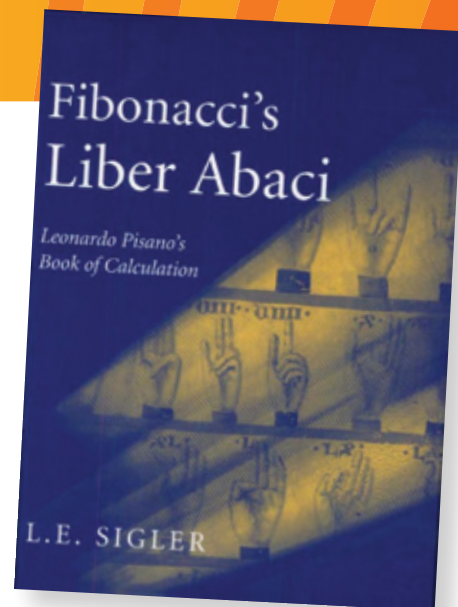
*Liber Abaci* - or the *Book of Calculation* - was a text written in 1202 by Leonardo Pisano, more commonly known today as Fibonacci, as a way to bring the mathematical methods of the Muslims in Africa and the Middle East back to Italy and Western Europe. It introduced such concepts as the Hindu-Arabic number system, along with place value (including the concept of zero), and many of the algorithms for performing arithmetic operations on these numbers to a society that was still using the Roman numeral system. *Liber Abaci* was a detailed text that taught how to use these concepts while providing proofs on why they worked, with many of these proofs based on the works of Euclid's *Elements*.

The first thing I noted was the way Fibonacci paired the history of the mathematical processes alongside the methods and proofs he used throughout the text; creating an engaging story as to where these methods came from, why they worked and how they could be applied. In today's society, so much of the "why" behind the way things work and where they came from has been lost to time – especially in the age of calculators, computers, and AI. We have become so efficient at solving arithmetic questions using algorithms and technology that we've forgotten

the story of how they came to be and why that is important to understand.

We've also lost the history of where many of these processes came from; many are now attributed to the mathematicians who expanded upon them, but not necessarily to the ones who first created them. Fibonacci's text gave credit where it was due. In our elementary and secondary schools now, much of this history has been lost or misattributed, and it is of great detriment to our students to erase many of the fascinating origins from where our numbers and operations originate. In fact, in Ontario, there has been a push to 'focus only on the numbers' in the curriculum by the Ministry of Education. I believe doing this not only leaves out so much rich storytelling from the mathematics being taught, but it also fails to allow for students from different and diverse backgrounds to see themselves represented in what can be a truly accessible field. So we begin to see math as an elite subject, accessible to only a gifted few, which is simply not the case.

Fibonacci's desire for writing *Liber Abaci* was not only to bring this "new" method of mathematics to just mathematical scholars, but also to the "common person" and provide them with methods of doing



numerical calculations in a far more efficient manner. In this he succeeded. Merchants carried these new methods with them on their travels and the methods quickly spread throughout the world. Through his works he was able to share the efficient and groundbreaking methods he learned from Muslim mathematicians with his peers back in Italy; and, little did he know, eventually with the rest of the world as well.

If you'd like to explore more, you can find Laurence Sigler's translation, (in case your 13<sup>th</sup> century Latin is as rusty non-existent as mine) with notes, published by Springer (2002) here: [archive.org/details/laurence-sigler-fibonacci-liber-abaci-2003](https://archive.org/details/laurence-sigler-fibonacci-liber-abaci-2003) ◊

*About the Author: Dr. Pamela Brittain is the Fields Institutes' K - 12 Academic Coordinator and resident math educator. Currently she is working on providing educators with courses that aim to 'demystify' mathematics and, like Fibonacci, she hopes to take her students past the how and into the why of many of the algorithms and methods we use to solve modern mathematical problems. If you'd like to check out these courses please visit [fieldsacademy.ca/eec/](https://fieldsacademy.ca/eec/) or email [pbrittai@fields.utoronto.ca](mailto:pbrittai@fields.utoronto.ca)*

# LIFE @ FIELDS

An insider's peek at 222 College St. life.



Longtime friends Messoud Efendiyev and Hermann Eberl bumped into each other at the Institute for a happy, impromptu reunion. It then took the most Fieldsian turn possible.

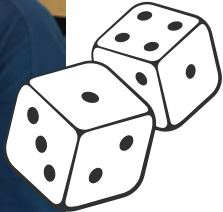


No visit to Fields is complete without a ferry ride on Lake Ontario. Thanks to our social committee for organizing a wonderful day at Toronto Island.





Why yes, we do like board games and snacks. Why do you ask?



Not these guys, though. They're still doing math.



Every now and then we're a biology research institute

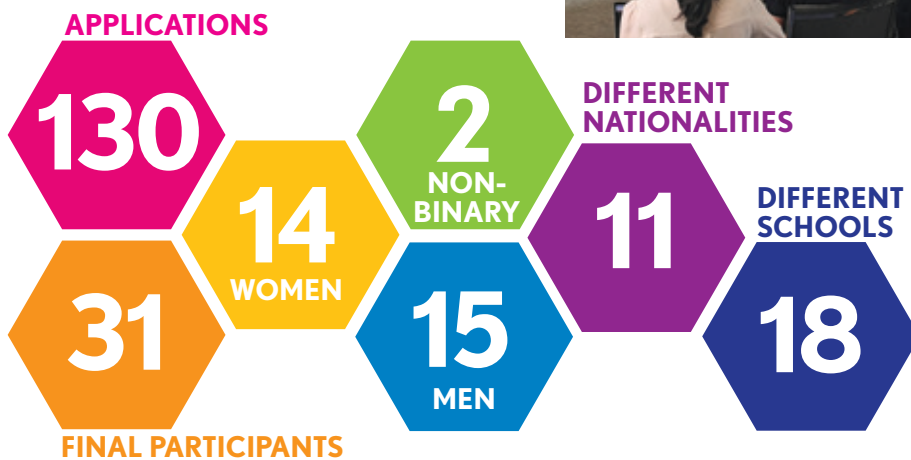
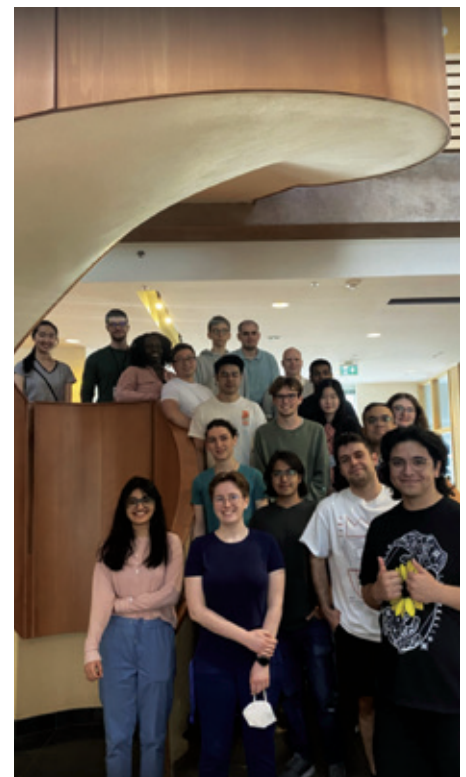
# SCHOOLS' IN FOR SUMMER

## Highlights from our 2023 Fields Undergraduate Summer Research Program (FUSRP)

Ran from:  
June 12 - August 9, 2023

### 8 Projects

- *On Field Extensions of Nonarchimedean Local Fields and Local Langlands Correspondence*
- *Active Human-Centric Evaluations in NLP Interpretability*
- *Statistical and Machine Learning and Applications*
- *Spectral Graph Invariants and Random Walks on Graphs*
- *Bootstrapping the Eigenvalues of Discrete Hyperbolic Surfaces*
- *Random Multiplicative Functions over Function Fields*
- *Spectral-Spatial Tissue Boundary Detection in Biomedical Hyperspectral Images*
- *Physiological Feature Extraction Using Facial Video Data*



Art  
Gallery of  
Ontario



Toronto  
Island

### IN THEIR OWN WORDS

The first impression I had was how talented everyone was and how different everyone's background was. As someone from a more applied math/engineering background, I found it really interesting when talking with the participants from more pure mathematics. As well, talking with people and realizing how similar our experiences were despite coming from different cultures was very cool.

James Zhu, FUSR 2023 alum

Not only did I meet incredible human beings from all around the world because of my own participation in this program, but I can also say that returning to supervise students this year was an incredible opportunity since I want to become a professional mathematician. This is the kind of experience you never forget.

William Verrault  
FUSR alum (2021, 2022), Supervisor (2023)



Niagara  
Falls



Do you know any students who would be interested in joining the next FUSR cohort?  
Visit our website to learn more.



# WIERINGA'S WALL LANDS AT FIELDS

## The passion and dedication of architecture student, Ross J. Cocks, has mathematically transformed the Institute's main lecture room



Last February, Fields staff received a message to come down to Room 230 for a mystery presentation. At the front of the room, fiddling with a series of papers and models on a Bristol board, was a young man with math tattoos running up both arms. He introduced himself as Ross J. Cocks, an architecture student at the nearby Daniels Faculty, and he had a proposition for the Institute.

Over the next 30 minutes, Ross would convince Fields that the building was missing a large, full-scale Wieringa Surface along the back wall of the main lecture hall and that he was the person who could solve the problem. Ross's presentation was solid. The math was accurate, the vision realized and it was clear he was deeply passionate about the project. He got a full blessing to go ahead.

On June 29, 2023, Ross unveiled his wall with a formal presentation of the mathematical and acoustic properties behind this architectural feat. In the weeks since it was installed, it's hard to imagine a time when it didn't adorn the room. If you've had the opportunity to attend a talk over the summer or fall, here's the story, in Ross's own words, of how our Wieringa's Wall came to be.

### THE "CURSE" OF PERIODICITY

The journey began with a paper by Trevor Cox and Peter D'Antonio, called "Schroeder Diffusers: A Review," which I was assigned to read for an unrelated project. In the paper I learned of an acoustic diffuser (the Schroeder Diffuser - more specifically - the Quadratic Residue Diffuser (QRD)) whose design was solely based-off mathematical formulas and principles found in number theory, and of which Schroeder himself theoretically connected to the optimization of acoustic diffusion.

While this seemed like the perfect design, there were drawbacks, namely, "[a] phase grating diffuser that exploits number theory, such as a QRD, is in many ways cursed by periodicity. A QRD needs periodicity to form even energy grating lobes, yet these lobes cause uneven scattering." The phrase "cursed by periodicity" really rang in my ear and I knew that there must be a design for a diffuser that is just as good as the QRD, but which is not weighed down by periodicity.

Architecture has a bad habit of poaching terminology and recasting it to fit its own discourse. Therefore, the first step was to look

at the real mathematical definition of periodicity and see what its "opposite" was. This is where I ran into the terms: randomness, non-periodicity and aperiodicity. All these terms were consolidated in the field of mathematical tiling. Both aesthetically and practically, this was a great area of mathematics in which to base my investigations, as tiling and architecture have a relationship that is as old as the disciplines themselves.

As I gained a deeper understanding of the definitions and principles aligned with the aforementioned definitions, I had a feeling that aperiodic tilings were going to be the

answer to the “curse of periodicity”. With this intuition, I began to see what architects had experimented with in the past. I was quickly referred to the incredible work done by Islamic architects, who worked with rotational symmetries in mosques and other structures that could not yield a periodic tiling, yet created an effortless sense of order and continuity.

But it wasn’t until I set my eyes on the iwan entrance to the Shah Mosque in Iran, with its muqarnas vaulting, that I had my “eureka” moment. I thought there must be a 3D surface that is aperiodic in nature, with which I could test its acoustic qualities and which would yield a result that was just as impressive as the QRD. I typed into Google: “3D Aperiodic Surfaces” and was only confronted with a handful of images. It was easy to spot my “golden goose” surface, wonderfully represented in sculpture by the artist Deborra Combs and computer scientist Duanne Bailey, as its form showed an order that could not be tamed periodically. It was from this point that I fell in love with the Wieringa Roof, and began to study it obsessively.

## DOING THE FIELDS WORK

The idea to implement a Wieringa Surface into an architectural setting came to me immediately after the acoustic results were completed. I had found a means to fabricate it, and when my conference paper was submitted, I thought to myself, “I have the research idea published, now it is time to realize it!”

However, I did not share the idea with my mentors right away. In my experience, I have found that in any serious architectural project, an idea is useless until you have a viable location and means. Therefore, I contacted Fields directly. I knew that if I found a location and a way of funding it, my superiors would be more likely to support the project.

I chose to partner with Fields because of my sincere love for the institution and the work that is done there. While I have been enrolled in many post-secondary mathematics classes, the bulk of my mathematical learning has been autodidactic. Fields’ free archive of YouTube seminars and lectures were an essential part of this learning process and I wanted to give back to the institution. I saw Wieringa’s Wall as an excellent way to do this.

When Fields said that it was a possibility and to gather together a proposal, I knew it was time to talk to my mentors. Thanks to the support and guidance of Daniels Faculty members Nicholas Hoban (Head of the Digital Fabrication Lab), John Nguyen and Dr. Brady Peters, the project became a reality. I also want to note the generosity of Dr. Marjorie Senechal, the Louise Wolff Kahn Professor Emerita in Mathematics and History of Science and Technology at Smith College, whose book *Quasicrystals and Geometry* became my mathematical bible for understanding aperiodic structures and the Wieringa Roof. When I contacted her via email in the summer of 2022, she answered promptly and worked with me throughout the project to ensure

the mathematics were sound and accurate.



### METHODOLOGY

Model created through a “cut and project method” using a 5D lattice to produce the vertices of a 2D Penrose tiling, then indexing each vertex through the levels of the cutting window, a rhombic icosahedron. The wall acts as an acoustic scattering surface, lowering the reverberation time of the room.

## PIECING IT ALL TOGETHER

What separates Wieringa’s Wall from other architectural projects is the sheer interdisciplinary demand. It does not have a single narrative, nor is it a piece that can be neatly cataloged into a series of concise narratives; it is a piece that produces a web of intricate relationships that are analogous to a reciprocal frame. Take one component away and the project ceases to support itself.

Before physical production began, I had to know the math, then construct a computational parametric model of the Wieringa Surface. I’m proud to say my model is the first of its kind. This was

also necessary to understand the acoustics and scale the Wieringa Surface computationally.

Several principles have to be outlined to understand the mathematics behind the project. They include:

## 1. Constructing Aperiodic Tessellations Through the Projection Method

*Note:* This is a well-documented mathematical method. In no way did I come up with this. My main sources for understanding it was from Dr. Marjorie Senechal's *Quasicrystals and Geometry* and N.G. de Bruijn's 1981 papers, *Algebraic Theory of Penrose's Non-Periodic Tilings of the Plane I & II*.

- a) Let  $E_n$  be the euclidean space of  $n$ -dimension. There exists an infinite discrete subspace  $\mathbb{Z}_n \subset E_n$ , such that for all  $z \in \mathbb{Z}_n$ ,  $z$  can be written as an  $n$ -tuple of integers. We will call  $\mathbb{Z}_n$  the integer lattice. Formally,  $\mathbb{Z}_n = \{z \in E_n: \{z_1, \dots, z_n\}, z_i \in \mathbb{Z}\}$ .
- b) For all  $z \in \mathbb{Z}_n$ , there exists a Voronoi Cell that is centered at  $z$ , denoted as  $V(z)$ . A *Voronoi Cell*,  $V(z)$ , is the set of all points in  $E_n$  that are closer to  $z$  than to any other  $z' \in \mathbb{Z}_n$ , for  $z' \neq z$ . For a regular lattice, which is what  $\mathbb{Z}_n$  is, the Voronoi cells for each lattice point are identical up to translation. Formally,  $V(z) = \{x \in E_n: \|x - z\| < \|x - z'\|, \forall z' \in \mathbb{Z}_n \setminus \{z\}\}$ .

- c) Let  $\mathcal{E}$  be a totally irrational  $m$ -dimensional subspace of  $E_n$ , that is, it intersects the  $n$ -dimensional integer lattice only at the origin, denoted as  $z_0$ , of  $E_n$ , with  $m < n$ , and let  $\mathcal{E}^\perp$  be the orthogonal complement to  $\mathcal{E}$ . Note that the dimension of  $\mathcal{E}^\perp$  is  $n - m$ . Now let  $\Pi: E_n \rightarrow \mathcal{E}$  be the orthogonal projection onto  $\mathcal{E}$  and  $\Pi^\perp: E_n \rightarrow \mathcal{E}^\perp$  be the orthogonal projection onto  $\mathcal{E}^\perp$ .



- d) If we take  $\Pi(\mathbb{Z}_n)$  we obtain a dense set in  $\mathcal{E}$  that is not going to be useful in the construction of an aperiodic set. We must therefore find a subset of  $\mathbb{Z}_n$  that will lead to a projection that has the properties we are looking for. We have to look outside the domain of  $\mathcal{E}$  for this since no lattice points, besides  $z_0$ , are included in  $\mathcal{E}$ .
- e) Thus, we look towards  $\mathcal{E}^\perp$  and build a compact subset  $w \subset \mathcal{E}^\perp$ , with a nonempty interior that captures the

needed properties. We will call  $w$  the *cutting window* of the projection. The simplest way to construct  $w$  is to take  $\Pi^\perp(V(z_0))$ , which creates an  $(n - m)$ -dimensional convex hull, which by definition is a compact proper subset of  $\mathcal{E}^\perp$ . The geometry of the cutting window depends on the dimensionality that we are dealing with.

- f) To prepare the set of points in  $\mathbb{Z}_n$  for the projection onto  $\mathcal{E}$ , we take points  $z \in \mathbb{Z}_n$ , such that  $\Pi^\perp(z) \in w$ , or lie within the cylinder  $C = w\mathcal{E}$ . Thus, we have constructed the set  $W = C \cap \mathbb{Z}_n$ .
- g) The process we have performed allows elements of  $W$  to be the vertices of an aperiodic tessellation on  $\mathcal{E}$ , after we take  $\Pi(W)$ .
- h) If two points  $z_1, z_2 \in \mathbb{Z}_n$  are neighboring vertices in the lattice, we can construct an edge between them. When we take  $\Pi(z_1)$  and  $\Pi(z_2)$  some of the information about the edge between  $z_1$  and  $z_2$  is preserved and it becomes an  $m$ -dimensional edge in  $\mathcal{E}$  between  $\Pi(z_1)$  and  $\Pi(z_2)$ .
- i) Take the union of  $W$  with neighboring vertices that have an edge with elements of  $W$ ; the resulting set partitions  $\mathcal{E}$  aperiodically, and is our tessellation of  $\mathcal{E}$ .



## 2. Constructing a Wieringa Roof Through The Projection Method

- a) There are three types of Penrose Tilings. All can be derived from one another and all are 2D, which means that our  $\mathcal{E}$  must also be 2D, that is  $m = 2$ . A good question (one that Deirdre asked, and one that I asked myself) is why do we need to use 5D space when projecting to 2D to get P3. Why couldn't it have been 4D or 8D? From my readings the reason comes from de Bruijn's paper.
- b) There are some important definitions that need to be laid out in order to understand the answer to this question. An infinite family of parallel hyperplanes in  $E_n$  with a fixed interplanar spacing of  $s$  is known as a *grid*. The vector with length  $s$  that is orthogonal to the hyperplanes is called the *grid vector*. The superposition of  $k$  grids is called a *multigrid*; thus, multigrids have  $k$  grid vectors with basis vectors  $\xi_1, \dots, \xi_k$ .
- c) In 1981 de Bruijn introduced the multigrid method for constructing the Penrose Tiling. The method came about by the observation that P3 created "ribbons" made of rhombs linked by parallel edges. By replacing the ribbons with straight lines, de Bruijn saw that there were 5 configurations of infinite parallel lines that were superpositioned to one another. Hence, the Penrose Tiling was



- d) This is logically equivalent to saying that the 2D Penrose Tiling could be constructed using a 5D space. We therefore have our  $n = 5$ .
- e) It follows that  $(n - m) = 3$ , which is the dimension of our  $\mathcal{E}^\perp$ . From the information gathered thus far, we know that our higher-dimensional integer lattice has a dimensionality of 5 (denoted as  $\mathbb{Z}_5$ ), our  $\mathcal{E}$  is a 2D object (a plane), and our  $\mathcal{E}^\perp$  is a 3D object (space).
- f)  $\mathcal{E}$  can be constructed using basis  $\xi = \{\xi_1, \xi_2\}$  where,
 
$$\xi_1 = \sqrt{2/5}[\cos(2\pi/5), \cos(4\pi/5), \cos(6\pi/5), \cos(8\pi/5), 1]T$$

$$\xi_2 = \sqrt{2/5}[\sin(2\pi/5), \sin(4\pi/5), \sin(6\pi/5), \sin(8\pi/5), 0]T$$
 $\mathcal{E}^\perp$  can be constructed using basis  $\xi^\perp = \{\xi_3, \xi_4, \xi_5\}$  where,
 
$$\xi_3 = \sqrt{2/5}[\cos(4\pi/5), \cos(8\pi/5), \cos(2\pi/5), \cos(6\pi/5), 1]T$$

$$\xi_4 = \sqrt{2/5}[\sin(4\pi/5), \sin(8\pi/5), \sin(2\pi/5), \sin(6\pi/5), 0]T$$

$$\xi_5 = \sqrt{2/5}[1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2}]T$$

$w = \Pi^\perp(V(z_0))$  is a rhombic icosahedron. With these ingredients you can construct a Penrose Tiling as outlined in the projection method.

- g) The set of vertices of the Penrose Tiling can be written formally as,  $P = \{p \in \Pi(W) : \{p_1, p_2\}, p_i \in \mathbb{R}\}$ . Note that all elements of  $P$  are 2D objects and therefore reside on a plane. In order to raise them into another dimension we must add another real number to the tuple. If we did this randomly, then we would create a surface which had little order in the added dimension.
- h) As mentioned in 2.6,  $w = \Pi^\perp(V(z_0))$  is a rhombic icosahedron where the points  $\Pi^\perp(z) \in w$  are projected into parallel four cross-sections of the rhombic icosahedron.

We can name these levels Level 1, ... , Level 4 (or simply 1,..., 4). It can be shown that these levels are precisely the values of the indices of the rhombs in the multigrid construction of P3.

- i) In de Bruijn's paper R.M.A. Wieringa suggested that if we assigned the level of each  $\Pi^+(z) \in w$  as a third dimension to its corresponding point in  $\Pi(w)$  such that we create a 3-tuple, we can create a corrugated surface that is made up of a single rhombus, and which itself is aperiodic in the added third dimension. The construction of the surface is very similar to the construction of Penrose's Tiling in that, if two points  $z_1, z_2 \in \mathbb{Z}_n$  are neighboring, the edge constructed between  $\Pi(z_1)$  and  $\Pi(z_2)$  is preserved as the extra dimension is appended according to the level that  $\Pi^+(z_1)$  and  $\Pi^+(z_2)$  fell on. In other words, the edges created for P3 are maintained as they rise into the third dimension. As before, when all the edges of neighboring vertices that fall within  $w$  are collected together and we take the union with  $w$ , the resulting set is the aforementioned surface. This surface described by de Bruijn and proposed by Wieringa was given the name Wieringa Roof, as there was thought that it could be used for it in architectural applications.

Recall that Wieringa Roofs are constructed using a single rhombus. This is the result of the information that is preserved from 5D space when

put through Wieringa's construction. Interestingly, the rhombus is significant in that its diagonals conform to the golden ratio. As such, it has been called the golden rhombus. You will find that the golden ratio is embedded in many ways within the Penrose Tilings as five-fold symmetry naturally incites these relationships. To go into detail would be a whole other article.

## A FEAST FOR THE EARS

In architecture, the prediction of acoustic performance is complicated by the interaction of numerous competing factors, including space dimensions, proportions, geometry, material properties and surface details. An important determinant of acoustical behavior is the design of architectural surfaces that affect the propagation of sound reflections through space. Such reflections may cause flutter echoes or comb filtering if they are not carefully controlled.

In proper acoustical design, sound reflections can be tamed by evenly dispersing the reflected sound energy. A key challenge associated with the design of effective surface diffusers is making sure that sound energy is dispersed uniformly throughout the sound field while keeping the sound energy within the room and not absorbing it. This maintains an appropriate reverberation time. Reverberation is the collection of the reflected sounds. Reverberation time, then, is the time after the source of the sound has ceased that it takes for the sound to fade away. The diffusion of sound is enhanced when every position in the diffuse sound field receives the same density of reflected

## HOW DOES IT SOUND?

If you look at the data collected before and after Wieringa's Wall was implemented at Fields, we see some great results (Figure 23)

1. The lower frequencies (63-125 Hz) are brought down below 1.0. This is great because we want all frequencies under 1.0!
2. Frequencies between 1-8 kHz significantly drop. This tells us that the geometry of the Wieringa Surface really quickens the reflection of these frequencies. However, not to an undesired reflection which would be below 0.5.
3. However the really significant result is the evening out of frequencies between 1-8 kHz. While the dropping of Reverberation Time is important, I am really excited with how it evened out a large sample of frequencies. If certain frequencies took a long time to reverberate while the others were too short the sound would become polluted and would give undesirable effects. In other words, the less messy of a graph, the less pollution in the sound. *Wieringa's Wall* evens out the 1-8 kHz frequencies into a really gentle slope. Beautiful.
4. Wieringa's Wall is not infallible, however, as 125-500 Hz peaks a little more than before. Thankfully, it is marginal and does not go above 1.0. Nothing that would affect the acoustics significantly.

All in all, a very successful installation.

## MATERIAL WORLD

- Expanded polystyrene (EPS) foam
- Formica®
- Contact Cement
- Acrylic Paint
- Silicon



energy. So, we are wanting to get an even broadband reception both in energy and time.

When designing a room for optimal acoustics, it is important to ensure that the reverberation time is appropriate for the intended use of the room. If the reverberation time is too long, the speech will be unintelligible and music will sound muddy. On the other hand, if the reverberation time is too short, the room will sound sterile and uninviting. By carefully considering the absorption characteristics of the materials used in a room, it is possible to achieve ideal reverberation times for any given application.

In practice, it is really hard to know the optimal, but there are a few guidelines; however, even these change from book to book. From my understanding, a room like Fields' 230 would need a reverberation time at about 0.5-1.0. So how do we calculate it? We use the metric Reverberation Time 60 (RT60). RT60 is defined as the measure of the time after the sound source ceases that it takes for the sound pressure level to reduce by 60 dB.

It can be difficult to put enough sound into a room to fully measure

RT60 directly, so we often extrapolate it using just a portion of the decay. If the time for the sound pressure level to decay by 20 dB is measured and multiplied by 3, we call our reverberation time a T20 measurement. If we measure the time for the sound pressure level to decay by 30 dB and multiply by 2, this is called a T30 measurement. In both cases, the measurement is begun after the first 5 dB of decay.

## REFLECTIONS ON A THESIS REALIZED

When I had my idea for the wall I was not nervous to share it, but I was wary that Fields would not take me seriously as an undergraduate, especially since I did not come from a Mathematics Department. Nothing could have been further from the truth. I was treated with respect and courtesy. My proposal was taken seriously. Very quickly, I felt like a contributing member of the Institute, rather than an outsider.

Wieringa's Wall was a taxing project that tested my limits many times. As expected, the week before the presentation was the busiest. I ended up only getting a total of 12 hours of sleep over 7 days. There were

many, many bumps and redirections along the way, including some dark moments. But thanks to the support of my mentors, Fields, my family (especially my wife, Stephanie) and fellow students who helped me put the final project together, I'm proud to say my dream was realized. Right now, I'm also finishing up two papers on the mathematics and the art behind the piece.

There's also a hope for positive interaction and illumination through this work. There seem to be so many negative feelings around where things are headed in the world and about the condition of humanity. When a visitor has the opportunity to interact with Wieringa's Wall I hope that all that negativity dissolves away, and that they become enthralled with the absolute beauty that is represented in that form. ◻

*Ross J. Cocks is currently working on his Master of Science in Pure Mathematics at the University of New Mexico. To learn more about his work, visit [rossjcocks.com](http://rossjcocks.com).*

# POSTDOC SPOTLIGHT: GAEL YOMGNE DIEBOU

The 2022-2023 Fields-AIMS-Perimeter postdoctoral fellow made inroads in his harmonic analysis and function spaces theory research, but still faced Toronto's most impossible math problem of all: affordable rent.

**Congratulations on wrapping up your postdoc at Fields. What will you be up to this fall?**

Starting in September, I will do a postdoc in the Department of Mathematics at the University of Toronto with Professors Catherine Sulem and Fabio Pusateri as supervisors. My position combines research and teaching activities for up to three years and my direction will definitely depend on my mentors. I will be exposed to research on the stability of solitons (traveling waves) or kinks in various dispersive PDEs models. In addition, I will potentially be working on the modulational theory of water waves (Dysthe equation), normal form transformations and related topics.

**What research did you work on at Fields?**

When I started my journey at Fields I had in mind two main problems, the first on the stability question for a priori global solutions of the harmonic maps heat flow in three dimensions and the second on the stability of stationary solutions to the Ericksen and Leslie model of nematic liquid crystal flow. I would like to describe the latter project.

The simplified Ericksen-Leslie's model describing the hydrodynamic flow of nematic liquid crystal.

**Is this the sort of outcome you were hoping for when you first received the Fields-AIMS-Perimeter postdoc?**

To be honest, I didn't start with any kind of precise picture of what the outcome would look like. It was more about taking things one step at a time, having daily goals, closing some gaps, advancing in my personal projects and those with collaborators and learning other mathematics from seminars and other sources. Whether or not it was the best option, this approach worked for me and there was no disappointment.

**Is your experience of Canada different from what you imagined?**

This is an interesting question, as the decision to come to Canada at the very first place was not easy to make. I knew Toronto in particular for its multiculturalism, diversity and high cost of living. My sister, already living and studying in Toronto, could also share valuable information with me. In this regard, I imagined meeting quite a number of people from different backgrounds, being exposed to a fairly liberal culture, having to allocate a big chunk of my salary for rent and groceries compared to my life in Germany. Those imaginings



turn out to be facts. However, regarding winter, I imagined a lot of negative experiences, but actually coped with it quite well.

**What were the highlights of your time at Fields?**


- A research paper I worked on and submitted when I started my position last year which has now been accepted to Journal of Functional Analysis.
- The meetings I had with Catherine Sulem and Fabio Pusateri were really valuable to me.
- The conference I attended at the CMS Winter Meeting organized by Prof. Sulem and Prof. Pusateri. I learned more of what my mentors are doing and I came across some nice people.
- Finally, there was a day I came to the office and had to work on something which needed my full attention over a long period and didn't have time to get lunch. By the time I finished, it was raining and I was hungry and tired. I went to the kitchen to drink the juice I kept in the fridge. When I opened the fridge, I saw a delicious-looking salad with leftovers of chicken leg quarters in

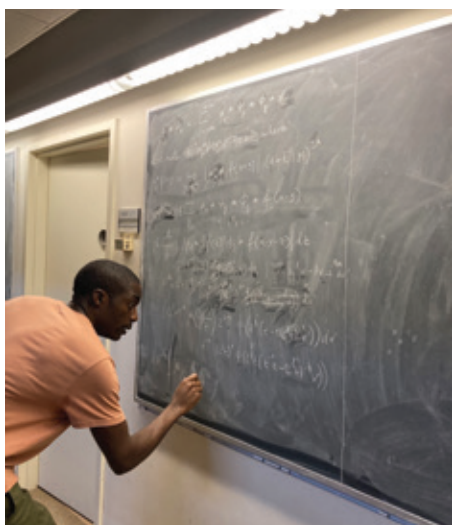
a box on which it is written EAT ME! What a relief! I am grateful to the Institute for those seemingly little things like food, which brings people together and helps fuel brains for more productive research.

### What advice would you give any future postdocs coming to Fields?

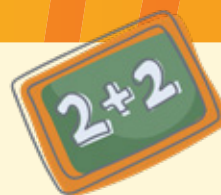
Definitely to take full advantage of all the available resources, like money to travel for conferences, the broad mathematical diversity at Fields and affiliated universities, the working environment. And take opportunities whenever they come your way!

### What's your favourite Fields story?

I want to highlight a repeated action which has been part of my days at Fields. More often than not, when I took the stairs to my third-floor office each morning, I would see an open office door. Behind the desk there was [Fields Liaison] Mrs. Miriam [Schoeman] who would lift up her head, smile and wave at me. This, for sure, I will miss! 



*In each issue, we will spotlight a postdoc researcher and their work.*



## Elementary Education Mathematics Programs

A strong math foundation for students begins with a strong math foundation for teachers.

Based on research and developed by an Ontario Certified teacher with a PhD in Math Education, the Fields Academy offers foundational math courses for elementary educators that help them build their confidence and knowledge in mathematics.

The courses are based on, and linked to, the Ontario Elementary Math curriculum.

### ASK A MATHEMATICIAN and FIELDS TRIPS

The **Ask a Mathematician** and **Fields Trip** programs aim to bring mathematicians directly to students through in-person and virtual visits.

The visits are free and focus on exploring mathematics not found in the standard Ontario curriculum.

Both programs provide elementary and secondary students the chance to interact with those who study and use math in their daily lives.

They also get to learn about possible careers in mathematics and where a degree in math can take them.

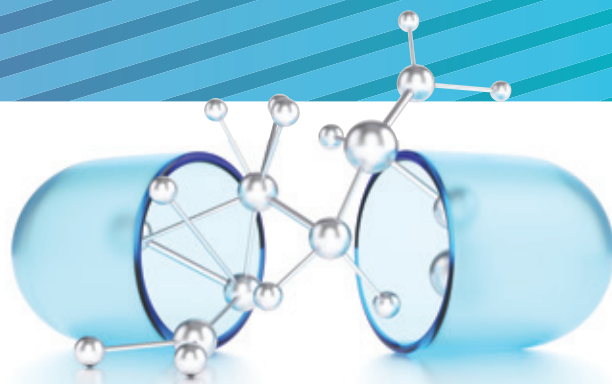
### GET IN TOUCH

Contact: **Dr. Pam Brittain**  
Academic Coordinator, K-12:  
[pbrittai@fields.utoronto.ca](mailto:pbrittai@fields.utoronto.ca)



# AI FOR GOOD

## Revolutionizing medical diagnostics with deep learning



In 2015, Abraham Heifetz, a computational biologist from the University of Toronto, raised \$6 million in seed money for his biomedical start-up, Atomwise. Heifetz had the bold ambition of using deep learning to analyze molecules found in millions of different medications already on the market and determine how well the molecules would “stick to a target disease” and ultimately switch it off with the fewest side effects. The idea was to reduce the time it would take to “discover” medicines by finding alternative uses and effective ways to treat disease from existing medicines – much like Aspirin was found to double as a treatment for managing heart disease.

Investors recognized the company’s potential when, during the 2014 Ebola outbreak in West Africa, Heifetz and his team used the technology to identify two existing medications that were found to significantly reduce the virus’ infection rate. They moved to clinical trials within four months of the discovery process. The speed was nearly unheard of at the time, and suggested, in a major global event like a pandemic, there were now possible avenues to reduce cost and treatment wait-time. Today, Atomwise is a thriving company based in Silicon Valley that combines convolutional neural networks with massive chemical libraries to discover new small molecule medicines.

In all the hype surrounding AI applications, healthcare diagnostics remains an area with the greatest promise for social good. With capabilities spanning from enhanced cancer detection to mapping individualized drug outcomes for each patient’s unique biome, the integration of artificial intelligence (AI) into healthcare diagnostics offers pivotal opportunities to save lives.

With that said, there are a number of hurdles to clear before we see anything close to widespread adoption of these technologies. Entrusting one’s health to an algorithm is still a risky decision, and there needs to be better communication from both engineers and medical professionals about what

is going on under the hood. As mathematicians, we can help translate knowledge the way we know best: through a technical deep-dive into the numbers.

### I SPY WITH MY AI: MEDICAL IMAGING AND OTHER SUCCESSES

X-rays, MRIs, CT scans, nuclear imaging, microscopy, and ultrasounds – these imaging techniques are cornerstones of modern medicine. Historically, these types of datasets have been difficult to work with. The dimensionality of these datasets is massive, while researchers are often limited by only a handful of labelled examples for statistical models to learn from.

As with other areas where AI is creating advances, deep learning techniques are pushing the boundaries of what is possible: Data is becoming more available, while algorithms are becoming more “label-efficient,” meaning they don’t need as much data to be trained effectively.

Traditional breast cancer screens, for example, consist of a set of 2D X-rays manually reviewed by a radiologist. The usefulness of these mammograms depends entirely on the diligence of the expert reviewer to recognize signs of cancer. Deep learning algorithms are being used today to consume datasets of images and learn to detect subtle patterns and anomalies. Some algorithms also incorporate patient-specific 3D imaging, and X-ray physics to improve fidelity. For example, a company called ScreenPoint Medical received FDA approval for a screening tool which has been shown to reduce radiologist workload by 44% without any drop in the recall rate of cancer detection.<sup>[i]</sup> Other companies have AI-driven, FDA-approved products for helping detect digital pathology (Paige AI), intracranial hemorrhage (MaxIQ AI), or general radiology (Aidoc), MRI (Icometrix, Subtle).<sup>[ii]</sup>

Here’s how the math works in the backend.

## CONVOLUTION NETWORKS

The convolutional layer is designed to adaptively learn spatial hierarchies of features from input images. When convolution was first invented, researchers would “hand-craft” the convolutional filter to detect pre-defined patterns in images. Today, the kernel or filter  $K$  is not just a fixed matrix; it contains weights that the network learns during training.

Given an input tensor  $I$  and a kernel tensor  $K$ , the convolution operation is defined as:

$$F(x, y) = \sum_{i=1}^m \sum_{j=1}^n I\left(x - i + \frac{m+1}{2}, y - j + \frac{n+1}{2}\right) \cdot K(i, j)$$

Here  $K$  is a kernel of dimension  $m \times n$  with learned weights such that when this filter is convoluted with an input image  $I$ , it produces a feature map  $F$  that emphasizes certain features (e.g. edges) in the input. In practice, this feature map becomes one layer; in order to combine multiple layers together, an addition of the activation function is required.

*Activation function:*

After the convolution, an activation function is applied, introducing non-linearity. This allows for stacking of multiple layers (as otherwise the stacking operation would simply reduce to another linear operation). For example, the ReLU function (denoted as  $R$  below) can be composed with the convolutional layer:

$$F'(x, y) = R(F(x, y))$$

By wrapping the convolutional filters with the activation function, multiple layers of the operation  $F'$  can be combined together, allowing the network to express increasingly complex patterns;  $n$  layers of the combination of convolutional - activation can be stacked as:

$$F'_n \circ F'_{n-1} \circ \dots \circ F'_1(x, y)$$

*Pooling layer:* The pooling layer is used to reduce the spatial dimensions of the feature map, retaining the most important information. The most common pooling operation is max-pooling, defined as:

$$P(u, v) = \max_{a=0}^s \max_{b=0}^t F'(u \times s + a, v \times t + b)$$

where:  $s \times t$  is the size of the pooling window.  $P$  is the pooled feature map.  $F'$  is the feature map after applying the activation function.

The fully connected layer is a traditional multi-layer perceptron that uses a softmax function for classification. Given a flattened feature map  $F''$  of size  $d$  (obtained after pooling, and then ‘flattening’ the resulting tensor into a single dimension), and weight matrix  $W$  of size  $d \times c$  (where  $c$  is the number of classes), the output  $O$  of the fully connected layer is:

$$O = \text{Softmax}(F'' \cdot W + b)$$

where  $b$  is the bias vector of size  $c$

*Softmax:* Finally, the Softmax function is defined as:

$$\text{Softmax}(z)_i = \frac{e^{z_i}}{\sum_{j=1}^c e^{z_j}}$$



The fully connected layer's weights  $W$  and biases  $b$  are learned during training to map the features extracted by the convolutional and pooling layers to the correct class labels. The output vector  $O$  contains the probabilities for each class, and the class with the highest probability is the predicted class for the input image  $I$ .

## GETTING AHEAD OF EARLY DIAGNOSTICS

**Melanoma Screening with Transfer Learning:** Most of the applications mentioned above have employed custom-built models, but interestingly, the concept of model pre-training and transfer learning is finding applications in healthcare. For example, researchers were able to take a general-purpose image classifier and fine-tune it into the engine for a skin-cancer screening test. In this study, researchers took the ResNet-50 model (a convolutional neural network with 50 layers) which was pretrained on the ImageNet dataset (a massive open-source image database consisting of over 100,000 concepts each represented by 1000 images). They fine-tuned an ensemble of classifiers on a set of skin cancer images, biopsy-verified, and created a hybrid classifier in which a dermatologist's decision would be augmented with the output of the automated classification result. Across 117 dermatologists, the fusion method achieved significantly improved detection versus the physician classification alone – 89% sensitivity against 66%, and 84% specificity against 62%.<sup>[iii]</sup>



**Edge Devices and Early Diagnostics:** The proliferation of wearable technology, such as the Apple Watch 8 and the Oura Ring 3, exemplifies the potential of personal health devices. These devices, equipped with advanced sensors and AI algorithms, can continuously monitor vital signs, sleep patterns, and other health metrics. For instance, the Apple Watch 8, with its enhanced heart rate monitoring and oxygen saturation detection, can potentially identify early signs of cardiac anomalies or respiratory disorders – a claim supported in the New England Journal of Medicine.<sup>[vii]</sup> Similarly, the Oura Ring 3, a 4g ring-shaped

wearable sensor, has been proven to provide high validity in the measurement of heart rate and heart rate variability compared to a gold-standard electrocardiogram.<sup>[viii]</sup> Such miniaturized devices are therefore extremely useful in long-term health monitoring in place of traditional clinical assessments, and are easily integrated with other AI-apps to deliver health insights such as sleep scoring with the help of machine-learning algorithms.<sup>[ix]</sup> In future, it is certain that edge devices such as these will be approved for a wider array of medical alerting, lifestyle coaching, or health monitoring applications. They represent the future of proactive healthcare, where early detection can lead to timely interventions, preventions, and improved patient outcomes.

Let's take a look at how these approaches can be expressed mathematically, and some surprising similarities between them.

**Model Distillation:** model distillation involves training a smaller model (i.e., a model with fewer trainable parameters, thereby making it more efficient) to attempt to replicate the behaviour of a larger model. This is typically done to reduce the computational footprint of

a model, e.g. to allow it to run on an 'edge' device such as a smart watch. Let's take a look at this process:

**Training the teacher model:** The process begins with training a large neural network, set up for optimal predictive power with less regard for the computational footprint of the model;  $M_t$  (teacher model) is trained on a dataset  $D_t$  to minimize a loss function  $L_t$ , optimizing over some model space  $M$ . Usually, the labelled data  $y$  is produced by humans (e.g. human-labelled diagnostic data), and the size and performance of  $M_t$  are limited by the scale of the dataset  $D_t$

$$M_t = \underset{M \in \mathcal{M}}{\operatorname{argmin}} L_t(y_t, M(D_t))$$

**Distilling Knowledge to the Student Model:** The next step involves training a (usually) smaller model  $M_s$  (student model) on the outputs of the teacher model  $M_t$ . Interestingly, the learner model is not limited to labelled data curated by humans, as it is much cheaper to produce



an output of the model  $M_t$  on some unlabelled datapoint. Therefore, its performance can potentially be assessed on a much larger dataset, and  $M_s$  typically optimized to be as small as possible in size while remaining close to the predictions of the teacher model  $M_t$

$$M_s = \underset{M \in \mathcal{M}}{\operatorname{argmin}} L_s(M_t(D_s), M(D_s))$$

**Transfer Learning:** Transfer learning involves using a pre-trained model on a source task as a starting point and fine-tuning the same model for some specific task.

1. Pre-training a model: As with distillation, the process begins with training a large neural network  $M_p$  on a source dataset  $D_p$  to minimize a loss function  $L_p$ ; unlike distillation, where the case is often that the dataset  $D_s$  is much larger ( $D_s \gg D_t$ ), it is always the case that the pre-training dataset is much larger than the fine-tuning dataset  $D_f$  ( $D_p \gg D_f$ ). The labelling for  $y_p$  is often not done explicitly, instead finding some training task that can be performed on unlabelled data (e.g. attempting to fill in masked “patches” in input images), making this pre-training process effectively unsupervised.

$$M_p = \underset{M \in \mathcal{M}}{\operatorname{argmin}} L_p(y_p, M(D_p))$$

2. Fine-tuning the pre-trained model: Using the pre-trained model  $M_p$  as the initial starting point, the model is fine-tuned on a smaller dataset  $D_f$  to minimize a loss function  $L_f$ . The model space here is the space of models  $M_p$ , of models that are optimized using the learned parameters (weights) of  $M_p$  as a starting point

$$M_f = \underset{M \in \mathcal{M}_p}{\operatorname{argmin}} L_f(y_f, M(D_f))$$

This approach is often more powerful than simply training the model  $M_f$  from scratch, as the model  $M_p$  trained on large corpuses of data provides a good starting point having learned the general dynamics of the data type  $M_p$  was trained on. For example, this paper in Nature <sup>[xiv]</sup> shows the potential of models pre-trained on general computer vision tasks being used in the medical domain.

Both techniques leverage knowledge from a pre-trained model. In distillation, the teacher model is pre-trained, and in transfer learning, the source model is pre-trained. Both involve the transfer of knowledge, with the student model replicating the training task directly, while in transfer learning the fine-tuned model is typically solving an “adjacent” task. These techniques, and many others, are driving progress in other fields of AI, and show great promise as they are increasingly being adapted in the field of healthcare.

## PUSH FOR PROGNOSIS

With so many AI resources allocated toward marketing, commerce and entertainment applications, it is important to remember that there are a number of motivated, skilled researchers putting everything they have into improving health outcomes for people.

As the applications improve, it is exciting to think of the lives that could be saved by more efficient, individualized diagnostic capacity. And behind every device is a mathematical foundation. More validation that the work mathematicians do can literally save lives. ◻

[i] [https://www.thelancet.com/journals/lanonc/article/PIIS1470-2045\(23\)00298-X/fulltext](https://www.thelancet.com/journals/lanonc/article/PIIS1470-2045(23)00298-X/fulltext)

[ii] <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=9103969>

[iii] [https://www.ejancer.com/article/S0959-8049\(19\)30427-7/fulltext#appsec1](https://www.ejancer.com/article/S0959-8049(19)30427-7/fulltext#appsec1)

[iv] <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=8588324>

[v] <https://www.siemens-healthineers.com/digital-health-solutions/ai-rad-companion>

[vi] <https://www.gehealthcare.ca/en-ca/products/edison>

[vii] <https://www.nejm.org/doi/full/10.1056/nejmoa1901183>

[viii] <https://iopscience.iop.org/article/10.1088/1361-6579/ab840a>

[ix] <https://www.mdpi.com/1424-8220/21/13/4302>

[xiv] <https://www.nature.com/articles/nature21056>

Alik Sokolov is Managing Director, Machine Learning, at the University of Toronto's RiskLab and co-founder/CEO of AI-driven investment start-up, Responsibli.

Daniel Jacobs is a PhD candidate in Neural Engineering at the University of Toronto.

# MULTIPLIER PROFILE: STRATOTEGIC INC.

## Connecting the world through nonparametric learning



Jeremy Henderson spent over two decades as a military pilot in the Canadian Air Force, with multiple overseas deployments and years coordinating global air force operations. During his time in the skies, he learned more than a few things about airspace and navigation systems and grew fascinated by ways to improve them. After leaving the military in 2019, Henderson consolidated his niche expertise through Stratotegic Inc., with the goal of assisting high-altitude platform station (HAPS) companies with flight and air traffic control coordination on a global scale.

Navigation is one of the biggest problems facing the HAPS aerospace industry. For instance, how do you navigate in an area you don't know much about? How do you make decisions for flight direction around a factor as complex as the wind? And how do you account for a multitude of unpredictable circumstances in high-conflict areas or post-natural disaster? There had to be technologies that could mitigate risk, reduce the chance of human error and deploy communications to underserved areas.

Henderson visualized methods to improve on networks of autonomous high altitude balloons. He would build upon

proven concepts of stratospheric communications that were limited by imprecise navigation of winds. If you can develop an AI system that comprehensively understands stratospheric winds, then you have essentially harnessed the wind. The balloons would arrange and keep themselves physically situated above an area where high-speed internet is required for a temporary amount of time, like after an earthquake that knocks out all power, or a war zone where the means of communication have been compromised. The AI algorithms could also be used to manage the position of drones or underwater devices.

### TAMING THE WIND

Temporary networks are not a new idea. Scientists began experimenting with "balloon communication" back in the early 20th century, using enormous, helium-filled balloons to carry radio transmitters into the atmosphere. Later, meteorologists used weather balloons to collect information on atmospheric pressure systems and thus predict weather patterns. More recently tech giants such as Alphabet Inc. and SpaceX have invested in balloon technology to provide internet access to remote areas across Africa, South America and Oceania.

*"Had it not been for the funding support from the Fields Multiplier program and Mitacs, we would have remained unaware of the vast potential inherent in this area of mathematics to bring transformative changes to the HAPS industry. Martin Guay's engineering expertise in this field of study, combined with Donna Shukaris and Martin Croteau's exceptional business insights and product commercialization skills, were the linchpins that propelled us to the prototype phase of our technology."*

**- Jeremy Henderson**

A relatively small player like Stratotegic can win with these behemoths by solving the technology's biggest problem: wind currents. Wind currents are part of a very complex flow system that change constantly and are very hard to predict. Climate change only increases the complexity. This summer, we observed right at home how ground telecom infrastructure can be vulnerable to forest fires, especially in the Western provinces and Northwest Territories. The



latter had to resort to Canada Post for dispatching evacuation notices due to destroyed communication infrastructure.

Meanwhile, the Maritime provinces have been experiencing a marked uptick in hurricanes annually, resulting in the loss of significant ground-based communication. While satellite internet might seem like an alternative, its speed is often limited, and its capacity to connect to standard cellphones is restricted to SOS messages. Complementarily, HAPS can integrate seamlessly in the stratosphere with existing cellular infrastructure and can fill the communication void in disaster areas as well as remote regions.


The key lies in the intelligence behind keeping the balloons and their positions synchronized despite the weather. **And the tool for solving it is mathematics.**

## THE SKY IS NO LONGER THE LIMIT

Henderson partnered with Martin Guay, a Professor of Mathematics and Engineering at Queen's University. Guay connected him with Fields, where he learned about the emerging Multiplier program. Linking up with Fields, where Stratotegic could tap into our vast network of mathematicians and academic partners, made sense from a research perspective. For

the industry side, Multiplier principals, Donna Shukaris and Martin Croteau, have helped advise on everything from securing intellectual property protection for the company's control system to structuring relationships with international partners.

At present, Stratotegic is testing prototypes and forging international partnerships in the US and Europe. Later this fall, they are also starting a major multi-year government project that pertains to QKD quantum communications and at the same time currently undergoing an initial investment round aimed at gathering the necessary funds to take the company's development to a commercial level. The funding effort has already garnered considerable attention from investors, making him confident that it will be a resounding success.

Having coordinated HAPS airspace and approvals with over half the countries in the world for their clients, Stratotegic is well underway to being known as a key expert in the industry. We expect great things from this smart start-up and believe that, in this particular case, the sky is no longer the limit. 

## A MATHEMATICAL MATRIX

Stratotegic is approaching their research as an optimal control problem, essentially trying to identify functionals in real time in an efficient manner. The mathematics involved, then, are essentially the tools used for stability analysis, averaging analysis and all the complexities associated with time-bearing dynamics.

The main issue is having to rely on learning techniques that are data efficient. There are many learning techniques that require gigs and gigs of data. But for an application like this, when you have an environment that changes all the time, they have to rely on a nonparametric learning tool that learns from a functional perspective, instead of getting the data point and regressing against what we have and trying to optimize parameters all the time.

*Fields Multiplier is an initiative that provides support for mathematics-based R&D with a high potential for commercialization. Visit [fieldsmultiplier.ca](https://fieldsmultiplier.ca) to learn more.*

# MULTIPLIER PROFILE: FINANCIAL WELLNESS

## Helping employees set themselves up for a healthy retirement

Imagine you're a recent university graduate and you've landed your first full-time job. On Day One, you walk into the office full of nervous anticipation, ready to make a great impression. A human resources representative leads you through an orientation session crammed full of new information that you are expected to retain. At the end of the session, the rep hands you a stack of paperwork to fill out. There's the standard record-keeping documentation, a medical insurance file or two, then an entire section on retirement planning.

You, a 23-year-old at the start of your career, are being asked at that moment to select how much money you would like to retire on in approximately four decades. But it's a game of chance. There is a list of questions that require a level of financial knowledge, along with around 50 potential investments that you are asked to choose from in order to park your contributions. Long, complicated names written in jargon that nobody understands. The wrong choice could mean not having enough to live off. What could possibly go wrong?

Chuck Grace and Adam Metzler, who help run the NSERC-funded Financial Wellness Lab at Western University's Ivey School of Business,

have studied data from thousands of Canadians on everything from risk tolerance and savings patterns to spending habits. The Lab's goal is to improve financial resilience for Canadians through data-driven research.

The Financial Wellness team developed an AI-based wellness tool for Retired Registered Savings Plans (RRSPs) that can help recalibrate the HR retirement savings plan process for Canadians. The tool, which forms part of a broader package of services that large benefits companies can offer to employers, streamlines the onboarding process to help employees make optimized, rapid and informed decisions about their financial future. The Lab joined forces with Aligned Capital Partners (ACP) and Fields Multiplier to build the prototype and take the finished product to market. ACP is now in the process of integrating the application into their back office and will launch later this year.

### HOW IT WORKS

While the experience is calibrated to be as easy as possible for users, there are two types of sophisticated, mathematics-rich research that have gone into it. The first is a risk tolerance questionnaire that can take financial advisors up to 45 minutes



*"While the tool is designed to be simple to use, the math is quite sophisticated. The clients, the employees assume that we've done our homework. By working with world-class researchers at an academic institution like Western and the Fields Institute, we can create that trust. They know there isn't going to be any bias in our recommendations."*

**- Chuck Grace,  
Financial Wellness Lab  
(Western University)**

to complete in person, but can now be done in a matter of seconds.

### PREDICTING A RICH FUTURE

With the tool already launched and generating revenue in a record turnaround of eight months, Grace and Metzler are now exploring additional ways to help Canadians become financially resilient. With Canadians feeling the crunch of financial uncertainty and the weight of inflation, a better, more robust client experience that helps employees make smarter long-term financial decisions could not arrive at a better time. 

# OUTSIDE VOICES

## FULL CIRCLE: My journey through mathematics education

This article has been adapted from the 2023 Margaret Sinclair Memorial Award Lecture delivered by Lynda Colgan on September 23, 2023.

The title for this retrospective was inspired by a quote from T.S. Eliot's *Little Gidding*:

"We shall not cease  
from exploration,  
and the end of all  
our exploring  
will be to arrive where  
we started,  
and know the place  
for the first time."

After more than four decades as a mathematics educator, I have arrived back where I started—now finally able to see clearly, once again, through the eyes of my 5-year-old self in kindergarten class at Toronto's Bruce Public School.

Was it the stars aligning that led me down myriad roads less travelled, affording me the unique opportunities that led to a highly unconventional career as a mathematics educator? If so, I am grateful for the pathways and shepherds that those astral wonders brought to light.

I have been an elementary, secondary and middle school teacher, LOGO pioneer, computer consultant, K-12 district coordinator for math, newspaper columnist, TV scriptwriter, children's book author, professor, researcher and outreach enthusiast. I have been honoured by my students and peers, and I have been vilified in the press for daring to say, "Math should be fun."

But each path inevitably led me back to the place I started: marvelling at the awe and wonder that mathematics is capable of producing in the open, curious mind. I'd like to

share a few highlights, insights and lessons learned from my path to the Margaret Sinclair Memorial Award podium.

### THE FORMATIVE YEARS

My path was set that day in kindergarten. I just didn't know what form it would take. What I did know was that I was mesmerized by straight lines that turned into winding stripes around a cylinder. I would spend the rest of my career attempting to replicate the sensation it created in me that day. In Grade 5, as a result of the 'New Math' which radicalized curriculum after Sputnik 1's success for the Soviet Union, I learned how to count in bases other than 10, fascinated by the Maya, who used base 20, the Egyptians who used base 12, and the Babylonians who used base 60, wondering what they were counting beyond fingers and toes. In Grade 13, I had my debut as a magician, performing a 'trick' with a solution of decanedioyl dichloride in cyclohexane floated on an aqueous solution of 1,6-diaminohexane. The class was amazed as a long thread formed at the interface. I pulled it out as fast as it was produced, and with an exaggerated *Ta Da!* I explained that I had used the magic of chemistry to make a 'nylon rope.' It was official. This was it for me.



Photo credit: Garrett Elliot



My first *mathematical* experience... turning straight lines into winding stripes around a cylinder. Original artwork by award-winning children's author & illustrator, Peggy Collins.

## EARLY TEACHING YEARS AND CHALLENGING THE 'OLD SCHOOL'

I began my professional training at the University of Toronto Faculty of Education, where I prepared to be a mathematics, science and biology teacher for Grades 7 to 12. At my faculty, the mathematics education instructors were all conscripted from the University of Toronto school (UTS), an independent school for gifted students. It was not surprising that the focus at the time was supporting elite mathematics students and teaching us how to do so by working through problems from competitions like the Canadian Mathematical Olympiad.

For one assignment, we were charged with developing a lesson that addressed the age-old problem of why “two negatives make a positive.” Although I knew it was risky, I chose to focus my lesson on addition of positive and negative integers using a large demonstration number line and a paper-doll-like “Where’s Waldo-” popsicle-stick caricature who could walk forwards or backwards and be oriented to move right or left on the number line. In a course called *Teaching Elementary Mathematics*, I learned a little about children’s mathematical development and the notion of “physical knowledge in mathematics;” hence my lesson based on “acting out” the integers’ magnitude and direction.

The peers in my course were unimpressed and my instructor decried it as being “overly simplistic” and “not mathematical.”

After what had been, in my mind, a disastrous lesson demonstration, I exited class quickly, eyes lowered. In the hallway, I heard someone call my name. Mary, a classmate, stopped to tell me that she was inspired by my lesson and hoped to use it during her next field placement in a Grade 9 class at an inner-city high school where many students were working well below grade level. She added that she might even adapt the lesson so that the students themselves walked on a number line in the gym. Out of a graduating class of 78 new educators, Mary and I were two of five in total to be hired as secondary school teachers for the next school year. Thanks to her affirmation, and my own placement, I knew I was on to something and trusted that I would find others who understood the type of mathemagical experiences I hoped to create for students.

## MOVING TO THE OTHER SIDE OF THE DESK

I began my career at Sir Oliver Mowat C.I. in Scarborough, where I was mentored by forward-thinking Head of Department, Woody Sparrow. Woody encouraged us to include coin tossing, dice rolling, paper folding, outdoor lessons, like using homemade hypsometers to make indirect measurements outside on the football field, and calculators in our programs.

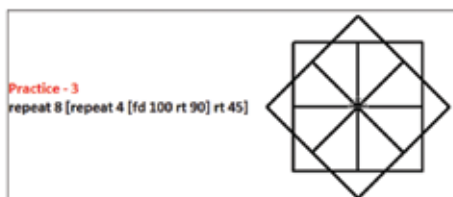
During this time, I was inspired to read *Mindstorms: Children, Computers, And Powerful Ideas*,

by Dr. Seymour Papert, and learn about why the LOGO turtle was challenging conventional implementations of technology in schools. Before LOGO, which is a programming language for teaching children mathematical concepts, technology was used in one of two ways: as a ‘tutor’ with children working through animated games camouflaged as nothing more than drill and practice of basic arithmetic facts and definitions; or, as a ‘tool,’ to sort data; perform calculations; or display text on a screen allowing it to be easily modified and formatted. With LOGO, the computer was the ‘tutee’ and the child used commands to program the movements of a ‘turtle.’

After challenging myself to ‘hard fun’ with the turtle (another expression coined by Dr. Papert to describe children’s intellectual fulfilment when their spinning squares emerged as a work of geometric art), I at last understood what my Flat-Stanley from my teacher preparation days and the turtle had in common—it was called *body-syntonic reasoning*—where students could understand, predict and reason about the turtle’s motion by imagining what they would do if they were the turtle. It was why Dr. Papert described the turtle as “giving the child a dual sense of accomplishment. When the turtle succeeded at what we told it to do – both because the child correctly instructed the turtle, and the turtle successfully carried out those instructions, the result of the latter the evidence of the former.”



To make a “total turtle trip,” (i.e., for the turtle to end in the same position in which it began), the LOGO turtle below needs to move FORWARD 50 steps, RIGHT TURN 90 degrees, FORWARD 50 steps, RIGHT TURN 90 degrees, FORWARD 50 steps, RIGHT TURN 90 degrees. This sequence of steps can be written as REPEAT 4 [FORWARD 50 steps, RIGHT TURN 90 degrees]. This “loop” shows the two steps that must be done four times to achieve the goal of the task.



This command asks the turtle to make one square [repeat 4[fd 100 rt 90]], next turn right 45 degrees and make another square, and repeat. The turtle will return back to its original position and orientation because by turning right 45 degrees eight times, the turtle will have moved through 360° or a complete circle.

LOGO itself, research about children’s learning with LOGO and the creative individuals drawn to be early-adopters of LOGO awakened in me a *maze of cognitive turbulence* (a term fellow LOGOphile Peter Skillen and I later used to name a conference), prompting me to: take up a teaching position in a Grade 5 inner-city school so that young children and I could become learning partners and do my best to concretize Dr. Papert’s credo:

*instead of trying to make children love the math they hate, why not offer them a math they can love.*

## THE JOY OF X

I had only been at the Faculty of Education at Queen’s for a few weeks before I met Ph.D. student Nathalie Sinclair, who is now a Research Chair in Tangible Mathematics at Simon Fraser University. Together with Dr. William Higginson, we developed an enrichment program for our prospective elementary and middle school teachers called *The Joy of X*. A group of about 65 teacher candidates volunteered to meet weekly to do math in small groups.

Over many weeks, the group met and played. Bill, Nathalie and I, as well as invited guests, often practiced the tasks together in advance and discussed the mathematics we hoped to point out. We used origami to explore maximum and minimum challenges and generate Dragon curves; learned about transformational geometry by creating Escher-like tessellations; demonstrated the power of exponential growth by modelling the story in *The Rajah’s Rice*; cut-out paper dolls to investigate the symmetries of frieze patterns and so much more. So disappointed were the students to see *The Joy of X* end, that Nathalie suggested creating a website to keep the young mathematics educators’ community alive. The SSHRC-funded *Connect-ME* website was a rich repository in which the new graduates could share resources for teaching creative

mathematics. While *Connect-ME* eventually waned due to lack of funding, many of the resources live on as part of the *Science Rendezvous Kingston* collection.

## MOVING THROUGH MEDIUMS

*The Joy of X* team approached Dr. Steve Lukits, then Editor of the Kingston Whig-Standard, to propose a weekly column about current education issues. My columns were always about mathematics and I wrote about everything from the math of Sudoku and the value of puzzles and games to support learning at home to pleading with parents NOT to help their children with their math homework, but instead asking them to ask their child to teach them the process that they were learning in school. Through the column, I celebrated our local mathletes and the excitement of the *Math Olympics*, going so far as to challenge readers to solve the problems posed and responded to reader requests for perspectives on why children just ‘played’ in kindergarten instead of learned their 1, 2, 3’s.

When Steve Lukits left the Whig-Standard to take on a professorship at The Royal Military College, the new editor, Christina Spencer offered me the opportunity of a lifetime—a full page, bi-weekly column about mathematics. Imagine the ‘hard fun’ that I had researching new topics and writing/illustrating pieces to educate and demystify mathematics. When I went to the

park, the grocery store or swimming pool, people would stop me and ask me questions about the most recent column. Educators were cutting out the columns and using the activities in their classrooms and requests were coming in for permission to reprint in other newspapers.

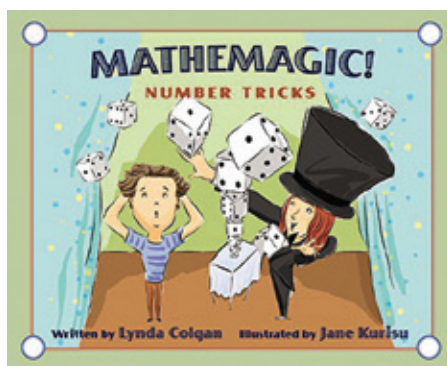
## THE MEDIUM IS THE MESSAGE

One day, I received a phone call from a producer of children's educational programs for TVO, suggesting that my column would be perfect to adapt into a television program for young children and their parents. I went to Ottawa to film a 'demo' tape, and with that *The Prime Radicals* came to be. The premise of each live-action episode was that math can be applied in everyday contexts. *The Prime Radicals* premiered on TVOKids, Saskatchewan Communications Network (SCN), CTV2 Alberta (Access) and Knowledge Network in January 2011. As of 2018 it's also airing on Starz Kids & Family in the US.

Two more educational television opportunities followed. The first, *MathXplosion*, was hosted by a real magician, Eric Leclerc. In each of these exciting, entertaining and funny math-shorts, 'mathemagician' Eric shared secrets from the not-so-hidden world of math, such as measuring the height of a tree using your thumb, using your 'head' as a referent for drawing a realistic person or testing the strength of a triangle by making a papier maché chair. Available in English and

French, *MathXplosion* won a *Youth Media Alliance* award for best digital content.

The popularity of the Whig-Standard articles also resulted in another 'spin-off' opportunity—the chance to write a children's book with *Kids Can Press*.



The cover of *Mathemagic! Number Tricks*

As my classrooms began to extend well beyond the walls at Duncan MacArthur Hall at Queen's University, through airwaves, libraries and *Mathemagic* webinars in schools across the country, other opportunities arose that sparked my passion and drove my energies.

## A RENDEZVOUS WITH SCIENCE

In 2010, I received an email from my colleague, Dr. Tom Russell, Physics Education Professor at the Faculty of Education, Queen's University that directed me to an article in the *Toronto Star* about an event that had occurred the day before—a festival called *Science Rendezvous*. Having been newly appointed the Director of the Mathematics, Science and Technology Education Group's Education Community Outreach

Centre, Tom thought that a program like the one in Toronto could meet the goals of the centre and he urged me to consider the possibility of having an event in Kingston.

Thus was born *Science Rendezvous Kingston* and its inaugural festival in May 2011. The event took place at the Faculty of Education's Duncan MacArthur Hall, where booths filled Student Street, the gymnasium and the Paul Park Garden. About 500 people attended and the response was so positive from volunteers and visitors that by the end of the day, plans were already in place for 2012.

Another milestone came in 2019. That was the year that saw *Science Rendezvous Kingston* win the STEAM BIG! Award for the best event in the country. Now firmly established as a very special annual event for families, *Science Rendezvous Kingston* set a new attendance record of 5200 visitors.

When COVID-19 hit in 2020, *Science Rendezvous Kingston* like so many events had to be cancelled in the name of public safety. It also meant that our much-anticipated 10<sup>th</sup> anniversary celebration needed to be put on hold. Though we were disappointed to cancel our 10<sup>th</sup> anniversary celebration, we began making preliminary plans for the future and managed very successful virtual, then hybrid events for the next few years.

By 2023, members of the public were hungry for live events, and we set a new attendance record: 5337 people visited *Science Rendezvous Kingston* to participate in more hands-on



experiments, demonstrations and displays than ever. *Science Rendezvous Kingston* is ‘in the books’ for 2024 and we are grateful.

## RETURNING TO THE PLACE I STARTED

Recently, the world of language education has been turned upside down. With the publication of the Right to Read by the Ontario Human Rights Commission, the worlds of education, psychology, linguistics and neuroscience have combined their expertise in the form of the Science of Reading. Language curriculum has experienced rapid and sweeping change, educators are revising instructional practices, parents are delighted to see a focus on explicit teaching strategies that can be extended to the home. Science, empirical research in separate domains can be integrated productively and collaboratively, with children as the most important beneficiaries.

The workshops following my Margaret Sinclair lecture were planned to celebrate not only my return to Kindergarten, but to spread the news about a new defining moment...one many more years in the making because of the silos of mathematics, mathematics education, developmental and cognitive psychology and neuroscience. Too long, our most important learners—those at the beginning of their mathematics learning trajectory have been the victims of the debate over who owns mathematics education.

Those in attendance at the Margaret Sinclair Memorial lecture program were privileged to have the Assessment and Instruction in Math (AIM) Collective team present, answer questions and address concerns. I am grateful to be involved in some small way, to finally shed light on why I was so excited about spatial sense some 65 years ago and why we need to start math education with the best teachers, the best materials and give them the right to learn math.



Lynda Colgan receives the Margaret Sinclair Memorial Award from Fields Deputy Director, Deirdre Haskell, on September 23, 2023.



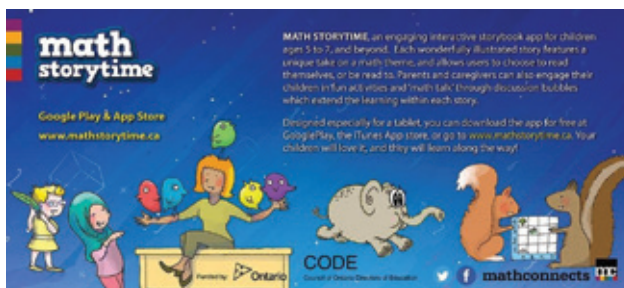
*The Prime Radicals.* Uncle Norm, Alana and Kevin in the workshop solving a conundrum about volume and angles in hockey.



*MathXplosion.* Magician turned mathematician, Eric Leclerc, teaching about probability, using socks, and why engineers use triangles in structures.



The Inspiring Your Child to Learn and Love Math, available in English and French.



The Math Storytime app, available in French and English



# POLL: CHALKBOARD, WHITEBOARD or DIGITAL PLATFORM?

We asked our friends, collaborators and the broader math community to chime in on which equation vehicle they prefer: a chalkboard, whiteboard or digital platform. Most importantly, we wanted to know why. While there was a clear winner (no big surprises here), the reasons were as varied, interesting and diverse as the mathematical community itself.

## TEAM CHALKBOARD

You can have an event without a blackboard, but it won't be a math event.

*Kumar Murty, Fields Institute Director*

I prefer chalk to whiteboards because chalk does not have a funny smell and I don't shudder about all the plastic waste that is being created. Also, chalkboards are generally much bigger than whiteboards, so you can leave lots of the lecture up for the students to read at their own pace.

*Deirdre Haskell, Fields Institute Deputy Director*

The sound while writing equations is unmatched.

*Farrukh Javed, Assistant Professor, Örebro University School of Business*

Mathematics is more beautiful when written in chalk. There is an unexplainable beauty to the way math appears on a chalkboard, much like the beauty of the Mona Lisa.

*Richard Ilemobade, Graduate Student, Obafemi Awolowo University*

Every other way of writing just feels wrong once you've used a chalkboard.

*Precious Itsuokor, Student, University of Lagos*

Blackboards have tactile feedback that makes it easier to write neatly. Whiteboard markers smudge easily and are unpredictable in terms of when they run out. Plus, marker fumes are more uncomfortable than chalk dust, although that's probably subjective.

*Steve McCormick, associate professor, Luleå University of Technology*

Chalkboards are the best mode for sharing ideas; they're cheap, convenient and durable solutions that allow mathematicians to write quickly, reliably and comfortably. Digital platforms are by far the most expensive solution and don't allow users to write as reliably and quickly as on a chalkboard. Some might argue that digital tools allow you to scan your work immediately, but it's so easy to just take a photo of a good old blackboard with your smartphone.

*Daniel Pont, Researcher, ETH Zurich*

Because I'm not an animal.

*Reese Lance, PhD candidate, UNC-Chapel Hill*

*Disclaimer: No Higonomo chalk was harmed in the making of this article.*

## TEAM WHITEBOARD

I don't like getting chalk dust everywhere and markers flow better. However, for my research most of the time I'm low-tech and use a paper notebook. And, in fact, I use pen because it flows better and keeps up with my thoughts. If something doesn't work I only strike a line through it in case I might want to look at it again.

*Nathan Carlson, Professor, California Lutheran University*

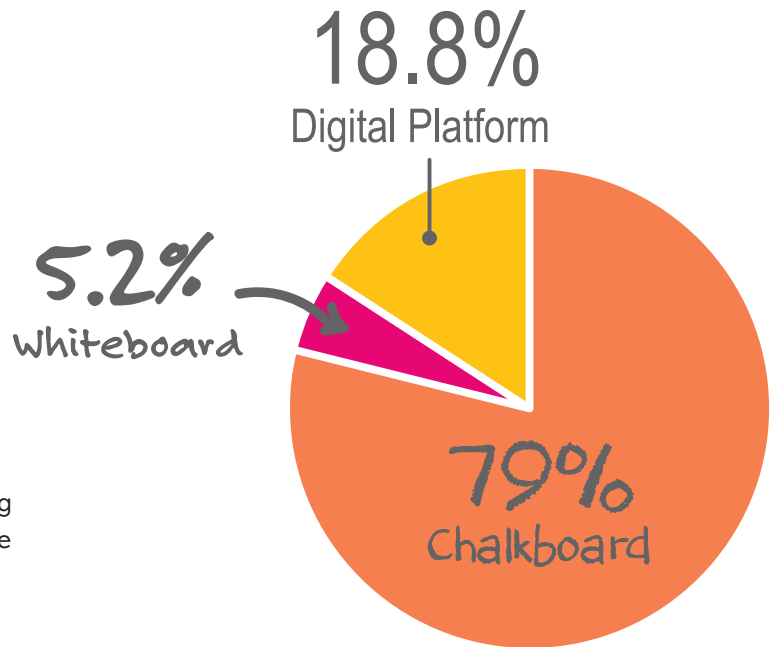
## TEAM DIGITAL PLATFORM

I used to prefer a whiteboard until we had Newline smart boards installed. I never knew how much time I spent erasing when I ran out of space until I had the technology to just save the page as a pdf and create a blank page to keep going!

*Amanda Davis, Professor, Forsyth Technical Community College*

More efficient means of distributing information.

*Stenio Musich, Brazil* ◊



# OUR 2023 FIELDS INSTITUTE FELLOWS

Created in 2002 to mark the Institute's 10th Anniversary, the designation of Fields Institute Fellow is awarded annually to a select group of people in recognition of their outstanding contributions.



## AYMAN CHIT

Ayman Chit currently serves as a Vice President at Sanofi Vaccines, a leading pharmaceutical company. He also holds the position of Assistant Professor at the University of Toronto's Leslie Dan Faculty of Pharmacy. With extensive expertise in the field, Chit leads teams of researchers and medical experts at Sanofi, driving the development of vaccines that align with global healthcare needs. At the University of Toronto, Chit actively collaborates with faculty members on research and teaching. Additionally, he plays a vital role in mentoring and advising graduate students, nurturing the next generation of scientific leaders and innovators.

Chit's primary research focus revolves around vaccine development and assessment. He is particularly interested in understanding the impact and burden of infectious diseases, and how to develop and deploy vaccines to counter this burden. Furthermore, his research extends to the study of the economics and administration of healthcare systems and the economics of drug and vaccine development and use.



## RICHARD KENYON

Richard Kenyon is the Erastus L. DeForest Professor of Mathematics at Yale University. His central mathematical contributions are in statistical mechanics and geometric probability. He established the first rigorous results on the dimer model, opening the door to recent spectacular advances in the Schramm–Loewner evolution theory. In most recent work, he introduced new homotopic invariants of random structures on graphs, establishing an unforeseen connection between probability and representation theory.



## JAVAD MASHREGHI

Javad Mashreghi is a Canadian mathematician and author working in the fields of functional analysis, operator theory and complex analysis. In particular, he is known for his contributions to analytic function spaces and operators acting on them. Mashreghi was the 35th President of the Canadian Mathematical Society (CMS), is a Lifetime Fellow of CMS, and works as Professeur Titulaire at Université Laval. He is immensely involved in various aspects of North America's mathematical community, having served on numerous editorials, administrative and selection committees across Canada and the U.S. (CMS, AMS, Fields Institute, CRM, NSERC, NSF). He is the Editor-In-Chief of the Canadian Mathematical Bulletin (2020–2025) and Concrete Operators (2018–2022), and an Associate Editor of the Proceedings of The American Mathematical Society (2020–2024). More recently, he became a Canada Research Chair in function spaces and a Fulbright Research Chair at Vanderbilt University.



### MESSOUD EFENDIEV

Messoud Efendiev is a world-renowned mathematical-biologist, leading scientist at the Helmoth Research Centre in Munich, member of the editorial boards of 10 international scientific periodicals, editor-in-chief of the International Journal of Biomathematics and Biostatistics, and author of over 160 scientific works and eight scientific monographs. He was James D. Murray distinguished professor at the University of Waterloo, York University, University of Toronto and Fields Institute, and is currently a distinguished professor at Western Caspian University.



### SYLVIA SERFATY

Sylvia Serfaty is Silver Professor at the Courant Institute of Mathematical Sciences of New York University. Prior to this she was Professor at the Université Pierre et Marie Curie (currently Sorbonne Université) at the Laboratoire Jacques-Louis Lions, and has held various appointments at the Courant Institute of NYU. She earned her BS and MS in Mathematics from the École Normale Supérieure in Paris in 1995, and her PhD from Université Paris Sud in 1999.

She works in calculus of variations, nonlinear partial differential equations, and mathematical physics. A large part of her work has focused on analyzing vortices in the Ginzburg-Landau model of superconductivity and on the statistical mechanics of systems of points with Coulomb-type repulsion.

She was the recipient of the European Mathematical Society prize in 2004, the Henri Poincaré prize in 2012 and the Mergier-Bourdeix Prize of the Académie des Sciences de Paris in 2013, and was a plenary speaker at the International Congress of Mathematicians in 2018. She was also named a Simons investigator in 2018 and elected to the American Academy of Arts and Sciences in 2019.



### CHRISTIAN GENEST

Christian Genest a professor in the Department of Mathematics and Statistics at McGill University, where he holds a Canada Research Chair. He is one of the leading statisticians in Canada, whose work has had dual impact on both theory and real-world applications. He is best known for his contributions to multivariate analysis and was a pioneer in the expansive use of copula models in science. Together with a few close collaborators, he combined nonparametric methods and the asymptotic theory of empirical processes to design a broad array of rank-based inference tools for building, selecting, fitting, and validating stochastic models within this class. Additionally, Genest has contributed to group decision making, prioritization techniques, multivariate extreme-value theory and, most recently, to space-time modelling of rare events in environmental science.

He is a recipient of the Statistical Society of Canada's Gold Medal for Research and was elected a Fellow of the Royal Society of Canada in 2015. Recently, Genest was awarded the 2023 CRM-Fields-PIMS Prize. ◻

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# FIELDS NOTES

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222 College Street  
Toronto, Ontario  
416-348-9710  
[www.fields.utoronto.ca](http://www.fields.utoronto.ca)

